Feedforward Neural Networks and Word Embeddings

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Outline

- Linear models
- 2 Limitations of linear models
- Neural networks
- A neural language model
- Word embeddings

LINEAR MODELS

Binary Classification with Linear Models

Example: the seminar at < time > 4 pm will

Classification task: Do we have an < time > tag in the current position?

Word	Lemma	LexCat	Case	SemCat	Tag
the	the	Art	low		
seminar	seminar	Noun	low		
at	at	Prep	low		stime
4	4	Digit	low		
pm	pm	Other	low	timeid	
will	will	Verb	low		

Feature Vector

Encode context into feature vector:

1 2 3	bias term -3_lemma_the -3_lemma_giraffe		1 1 0
102 103	2_lemma_seminar -2_lemma_giraffe		1 0
	 -1_lemma_at -1_lemma_giraffe		1 0
302 303	 +1_lemma_4 +1_lemma_giraffe		1 0
	•••	•••	

Dot product with (initial) weight vector

$$h(X) = X \cdot \Theta^{T} \qquad X = \begin{bmatrix} x_{0} = 1 \\ x_{1} = 1 \\ x_{2} = 0 \\ \dots \\ x_{101} = 1 \\ x_{102} = 0 \\ \dots \\ x_{201} = 1 \\ x_{202} = 0 \\ \dots \\ x_{301} = 1 \\ x_{302} = 0 \\ \dots \end{bmatrix} \qquad \Theta = \begin{bmatrix} w_{0} = 1.00 \\ w_{1} = 0.01 \\ w_{2} = 0.01 \\ \dots \\ x_{101} = 0.01 \\ x_{102} = 0.01 \\ \dots \\ x_{201} = 0.01 \\ x_{202} = 0.01 \\ \dots \\ x_{301} = 0.01 \\ x_{302} = 0.01 \\ \dots \end{bmatrix}$$

Prediction with dot product

$$h(X) = X \cdot \Theta^{T}$$

$$= x_{0}w_{0} + x_{1}w_{1} + \dots + x_{n}w_{n}$$

$$= 1 * 1 + 1 * 0.01 + 0 * 0.01 + \dots + 0 * 0.01 + 1 * 0.01$$

Predictions with linear models

Example: the seminar at < time > 4 pm will

Classification task: Do we have an < time > tag in the current position?

Linear Model: $h(X) = X \cdot \Theta^T$

Prediction: If $h(X) \ge 0$, yes. Otherwise, no.

Getting the right weights

Training: Find weight vector Θ such that h(X) is the correct answer as many times as possible.

- \rightarrow Given a set T of training examples $t_1, \dots t_n$ with correct labels y_i , find Θ such that $h(X(t_i)) = y_i$ for as many t_i as possible.
- $\rightarrow X(t_i)$ is the feature vector for the i-th training example t_i

Dot product with trained weight vector

$$h(X) = X \cdot \Theta^{T} \qquad X = \begin{bmatrix} x_{0} = 1 \\ x_{1} = 1 \\ x_{2} = 0 \\ \dots \\ x_{101} = 1 \\ x_{102} = 0 \\ \dots \\ x_{201} = 1 \\ x_{202} = 0 \\ \dots \\ x_{301} = 1 \\ x_{302} = 0 \\ \dots \end{bmatrix} \qquad \Theta = \begin{bmatrix} w_{0} = 1.00 \\ w_{1} = 0.001 \\ w_{2} = 0.001 \\ \dots \\ x_{101} = 0.012 \\ x_{102} = 0.0015 \\ \dots \\ x_{201} = 0.4 \\ x_{202} = 0.005 \\ \dots \\ x_{301} = 0.1 \\ x_{302} = 0.04 \\ \dots \end{bmatrix}$$

E.g. measure semantic similarity:

sim(time)
0.0014
0.0014
0.1
2.01
3.02
0.5

$$h(X) = X \cdot \Theta^{T} \qquad X = \begin{bmatrix} x_{0} = 1.0 \\ x_{1} = 50.5 \\ x_{2} = 52.2 \\ \dots \\ x_{101} = 45.6 \\ x_{102} = 60.9 \\ \dots \\ x_{201} = 40.4 \\ x_{202} = 51.9 \\ \dots \\ x_{301} = 40.5 \\ x_{302} = 35.8 \\ \dots \end{bmatrix} \qquad \Theta = \begin{bmatrix} w_{0} = 1.00 \\ w_{1} = 0.001 \\ w_{2} = 0.001 \\ \dots \\ x_{101} = 0.012 \\ x_{102} = 0.0015 \\ \dots \\ x_{201} = 0.4 \\ x_{202} = 0.005 \\ \dots \\ x_{301} = 0.1 \\ x_{302} = 0.04 \\ \dots \end{bmatrix}$$

$$h(X) = X \cdot \Theta^{T}$$

$$= x_{0}w_{0} + x_{1}w_{1} + \dots + x_{n}w_{n}$$

$$= 1.0 * 1 + 50.5 * 0.001 + \dots + 40.5 * 0.1 + 35.8 * 0.04$$

$$= 540.5$$

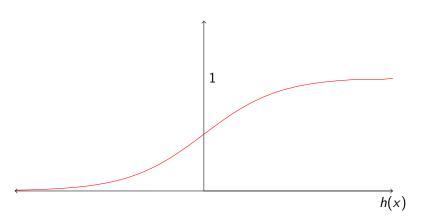
Classification task: Do we have an < time > tag in the current position?

Prediction: h(X) = 540.5

• Can we transform this into a probability?

Sigmoid function

We can push h(X) between 0 and 1 using a **non-linear** activation function The **sigmoid function** $\sigma(Z)$ is often used



Logistic Regression

Classification task: Do we have an < time > tag in the current position?

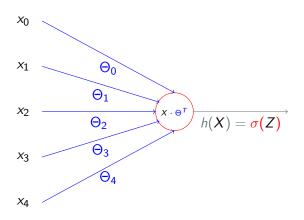
Linear Model: $Z = X \cdot \Theta^T$

Prediction: If $\sigma(Z) > 0.5$, yes. Otherwise, no.

Logistic regression:

- Use a **linear model** and squash values between 0 and 1.
 - Convert real values to probabilities
- Put threshold to 0.5.
- Positive class above threshold, negative class below.

Logistic Regression

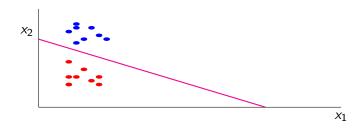


LINEAR MODELS: LIMITATIONS

Decision Boundary

What do linear models do?

- $\sigma(Z) > 0.5$ when $Z(=X \cdot \Theta^T) \geq 0$
- Model defines a decision boundary given by $X \cdot \Theta^T = 0$ positive examples (have time tag) negative examples (no time tag)



Exercise

When we model a task with linear models, what assumption do we make about positive/negative examples?

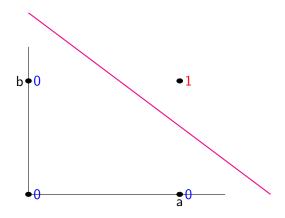
Modeling 1: Learning a predictor for \land

a	b	$a \wedge b$	b ∮ 0	• 1
0	0	0		
0	1	0		
1	0	0		
1	1	1	• 0	0

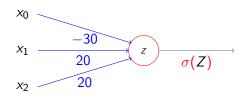
Features: a, b Feature values: binary

Can we learn a linear model to solve this problem?

Modeling 1: Learning a predictor for \land

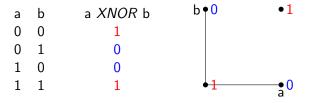


Modeling 1: Logistic Regression



<i>x</i> ₀	x_1	<i>x</i> ₂	$x_1 \wedge x_2$
1	0	0	$\sigma(1*-30+0*20+0*20) = \sigma(-30) \approx 0$
1	0	1	$\sigma(1*-30+0*20+1*20) = \sigma(-10) \approx 0$
1	1	0	$\sigma(1*-30+1*20+1*20) = \sigma(-10) \approx 0$
1	1	1	$\sigma(1*-30+1*20+1*20) = \sigma(10) \approx 1$

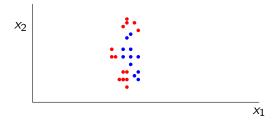
Modeling 2: Learning a predictor for XNOR



Features: a, b Feature values: binary

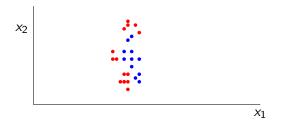
Can we learn a linear model to solve this problem?

Non-linear decision boundaries



Can we learn a linear model to solve this problem?

Non-linear decision boundaries



Can we learn a linear model to solve this problem?

No! Decision boundary is **non-linear**.

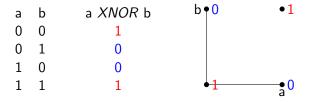
Learning a predictor for XNOR

Linear models not suited to learn non-linear decision boundaries.

Neural networks can do that.

NEURAL NETWORKS

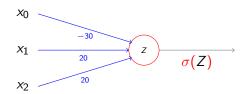
Learning a predictor for XNOR



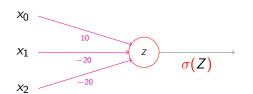
Features: a, b Feature values: binary

Can we learn a **non-linear model** to solve this problem? Yes! E.g. through function composition.

Function Composition

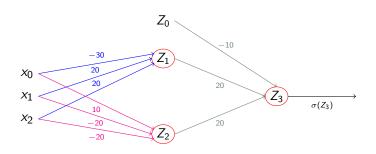


<i>x</i> ₀	x_1	<i>x</i> ₂	$x_1 \wedge x_2$
1	0	0	≈ 0
1	0	1	≈ 0
1	1	0	≈ 0
1	1	1	pprox 1



x_0	x_1	x_2	$\neg x_1 \wedge \neg x_2$	
1	0	0	≈ 1	
1	0	1	≈ 0	
1	1	0	≈ 0	
1	1	1	≈ 0	

Function Composition



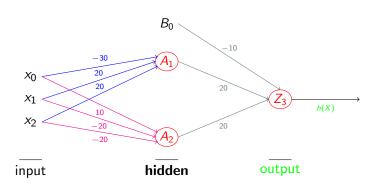
<i>x</i> ₀	x_1	<i>x</i> ₂	$\sigma(Z)$	Z_1) $\sigma(Z_2)$	$\sigma(Z_3)$
1	0	0	≈	0 ≈ 1	$\sigma(1*-10+0*20+1*20) = \sigma(10) \approx 1$
1	0	1	\approx	$0 \approx 0$	$\sigma(1*-10+0*20+0*20) = \sigma(-10) \approx 0$
1	1	0	≈		$\sigma(1*-10+0*20+0*20) = \sigma(-10) \approx 0$
1	1	1			$\sigma(1*-10+1*20+1*20) = \sigma(30) \approx 1$

Feedforward Neural Network

We just created a **feedforward neural network** with:

- 1 input layer X (feature vector)
- 2 weight matrices $U = (\Theta_1, \Theta_2)$ and $V = \Theta_3$
- 1 hidden layer **H** composed of:
 - ▶ 2 activations $A_1 = \sigma(Z_1)$ and $A_2 = \sigma(Z_2)$ where:
 - $\star Z_1 = X \cdot \Theta_1$
 - ★ $Z_2 = X \cdot \Theta_2$
- 1 output unit $h(X) = \sigma(Z_3)$ where:
 - $ightharpoonup Z_3 = \mathbf{H} \cdot \Theta_3$

Feedforward Neural Network



Computation of hidden layer H:

•
$$A_1 = \sigma(X \cdot \Theta_1)$$

$$\bullet \ A_2 = \sigma(X \cdot \Theta_2)$$

•
$$B_0 = 1$$
 (bias term)

Computation of output unit h(X):

•
$$h(X) = \sigma(\mathbf{H} \cdot \Theta_3)$$

Feedforward Neural Network

Classification task: Do we have an < time > tag in the current position?

Neural network: $h(X) = \sigma(\mathbf{H} \cdot \Theta_n)$, with:

$$\mathbf{H} = egin{bmatrix} B_0 = 1 \ A_1 = \sigma(X \cdot \Theta_1) \ A_2 = \sigma(X \cdot \Theta_2) \ & \cdots \ A_j = \sigma(X \cdot \Theta_j) \end{bmatrix}$$

Prediction: If h(X) > 0.5, yes. Otherwise, no.

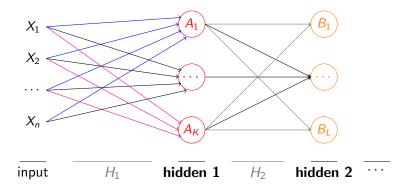
Getting the right weights

Training: Find weight matrices $U = (\Theta_1, \Theta_2)$ and $V = \Theta_3$ such that h(X) is the **correct answer** as many times as possible.

- \rightarrow Given a set T of training examples $t_1, \dots t_n$ with **correct labels y_i**, find $U = (\Theta_1, \Theta_2)$ and $V = \Theta_3$ such that $h(X) = \mathbf{y_i}$ for as many t_i as possible.
 - \rightarrow Computation of h(X) called forward propagation
 - $\rightarrow U = (\Theta_1, \Theta_2)$ and $V = \Theta_3$ with error back propagation

Network architectures

Depending on task, a particular network architecture can be chosen:



Note: Bias terms omitted for simplicity

Multi-class classification

- More than two labels
- ullet Instead of "yes" and "no", predict $c_i \in \mathcal{C} = \{c_1, \cdots, c_k\}$
- Not just <time> label but also <etime>,<\etime>,...
- Use k output units, where k is number of classes
 - Output layer instead of unit
 - Use softmax to obtain value between 0 and 1 for each class
 - ► Highest value is right class

A NEURAL LANGUAGE MODEL

Neural language model

- Early application of neural networks (Bengio et al. 2003)
- Task: Given k previous words, predict the current word Estimate: $P(w_t|w_{t-k}, \dots, w_{t-2}, w_{t-1})$
- Previous (non-neural) approaches:

Problem: Joint distribution of consecutive words difficult to obtain

- \rightarrow chose small history to reduce complexity (n=3)
- ightarrow predict for unseen history through back-off to smaller history

Drawbacks:

Takes into account small context

Does not model similarity between words

Word similarity for language modeling

- The cat is walking in the bedroom
- The dog was running in a room
- A cat was running in a room
- A dog was walking in a bedroom
 - → Model similarity between (cat,dog), (room, bedroom)
 - \rightarrow Generalize from 1 to 2 etc.

Neural Language Model (LM)

Solution:

Use word embeddings to represent each word in history

- → Each word is represented in relation to the others
- → Distributed word feature vector

Feed to a neural network to learn parameters for the LM task

Feedforward Neural Network for LM

Training example: The cat is walking in the bedroom

Neural network input:

Look at words preceeding bedroom

- \rightarrow The cat is₋₄ walking₋₃ in₋₂ the₋₁ bedroom
- \rightarrow Create word embedding (LT_i) for window

Give LT_i as input to Feedforward Neural Network

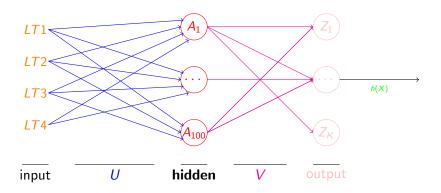
Neural network training:

Predict current word (forward propagation)

→ should be bedroom

Train weights by backpropagating error

Feedforward Neural Network for LM



Input: word embeddings LT;

Output: predicted label (current word)

Note: Bias terms omitted for simplicity

Feedforward Neural Network

```
Input layer (X): Word features LT1, LT2, LT3, LT4

Weight matrices U, V

Hidden layer (H): \sigma(X \cdot U + d)

Output layer (0): H \cdot V + b

Prediction: h(X) = softmax(0)
```

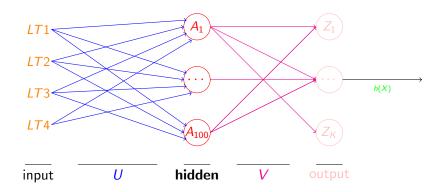
Predicted class is the one with highest probability (given by softmax)

Weight training

Training: Find weight matrices U and V such that h(X) is the **correct** answer as many times as possible.

- \rightarrow Given a set T of training examples $t_1, \dots t_n$ with **correct labels y**_i, find U and V such that $h(X) = y_i$ for as many t_i as possible.
 - \rightarrow Computation of h(X) with forward propagation
 - $\rightarrow U$ and V with error back propagation

Forward Propagation



Forward propagation:

 \rightarrow Perform all operations to get h(X) from input LT.

Forward Propagation

```
Input layer (X): Word features LT1, LT2, LT3, LT4

Weight matrices U, V

Hidden layer (H): \sigma(X \cdot U + d)

Output layer (0): H \cdot V + b

Prediction: h(X) = softmax(0)
```

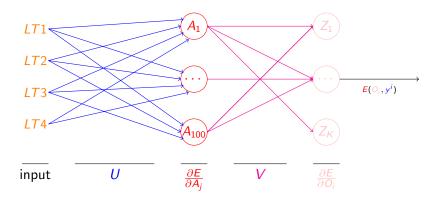
Predicted class is the one with highest probability (given by softmax)

Goal of training: adjust weights such that correct label is predicted

→ Error between correct label and prediction is minimal

Sketch:

- Convert difference between prediction and error into derivatives
- Compute derivatives in each hidden layer from layer above
 - ▶ Backpropagate the error derivative with respect to the output of a unit
- Use derivatives w.r.t the activities to get error derivatives w.r.t incomming weights



Backpropagation:

- \rightarrow Compute E
- \rightarrow Compute $\frac{\partial E}{\partial O_i}$

Compute error at output E:

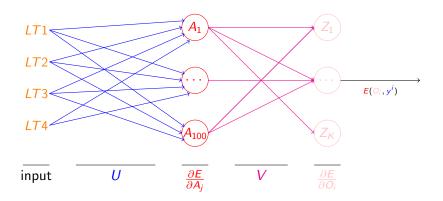
Compare output unit with y^i

 \triangleright y^i vector with 1 in correct class, 0 otherwise

$$E = \frac{1}{2} \sum_{i=1}^{n} (y_i - O_i)^2$$
 (mean squared)

Compute $\frac{\partial E}{\partial O_i}$:

$$\frac{\partial E}{\partial O_i} = -(y_i - O_i)$$



Backpropagation:

 \rightarrow Compute $\frac{\partial E}{\partial A_i}$

Compute derivatives in each hidden layer from layer above:

Compute derivative of error w.r.t logit

$$\frac{\partial E}{\partial Z_i} = \frac{\partial E}{\partial O_i} \frac{\partial O_i}{\partial Z_i} = \frac{\partial E}{\partial O_i} O_i (1 - O_i) \text{ (Note: } O_i = \frac{1}{1 + e^{-Z_i}})$$

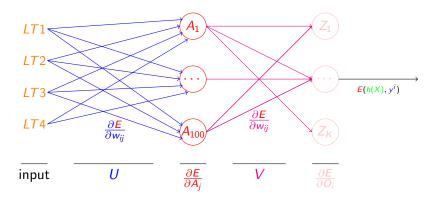
Compute derivative of error w.r.t previous hidden unit

$$\frac{\partial \mathbf{E}}{\partial A_j} = \sum_{i} \frac{\partial \mathbf{Z}_i}{\partial A_j} \frac{\partial \mathbf{E}}{\partial \mathbf{Z}_i} = \sum_{i} \mathbf{w}_{ji} \frac{\partial \mathbf{E}}{\partial \mathbf{Z}_i}$$

Compute derivative w.r.t. weights

$$\frac{\partial \mathbf{E}}{\partial w_{ji}} = \frac{\partial \mathbf{Z}_i}{\partial w_{ji}} \frac{\partial \mathbf{E}}{\partial \mathbf{Z}_i} = O_i \frac{\partial \mathbf{E}}{\partial \mathbf{Z}_i}$$

 \rightarrow Use **recursion** to do this for every layer



Weight training

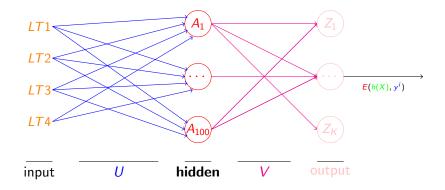
Training: Find weight matrices U and V such that h(X) is the **correct** answer as many times as possible.

- \rightarrow Computation of h(X) with forward propagation
- $\rightarrow U$ and V with error back propagation

For each batch of training examples

- Forward propagation to get predictions
- 2 Backpropagation of error
 - Gives gradient of E given input
- Modify weights (gradient descent)
- Goto 1 until convergence

Word Embedding Layer



Word Embedding Layer

• Each word encoded into index vector
$$w_i = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

- LT_i is dot product of weight matrix C with index of w_i
 - \rightarrow **C** is **shared** among all words

Dot product with (trained) weight vector

 $W = \{ \mathsf{the}, \mathsf{cat}, \mathsf{on}, \mathsf{table}, \mathsf{chair} \}$

$$w_{table} = \begin{bmatrix} 0\\0\\0\\1\\0 \end{bmatrix} \quad C = \begin{bmatrix} 0.02 & 0.1 & 0.05 & 0.03 & 0.01\\0.15 & 0.2 & 0.01 & 0.02 & 0.11\\0.03 & 0.1 & 0.04 & 0.04 & 0.12 \end{bmatrix}$$

$$LT_{table} = w_{table} \cdot C^{T} = \begin{bmatrix} 0.03\\ 0.02\\ 0.04 \end{bmatrix}$$

Words get mapped to lower dimension

 \rightarrow Hyperparameter to be set

Dot product with (initial) weight vector

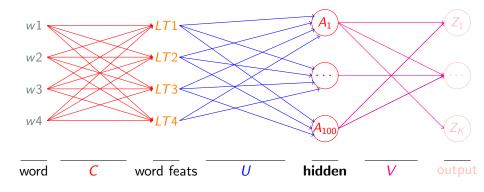
 $W = \{$ the,cat,on,table,chair $\}$

$$w_{table} = \begin{bmatrix} 0\\0\\0\\1\\0 \end{bmatrix} \quad C = \begin{bmatrix} 0.01 & 0.01 & 0.01 & 0.01 & 0.01\\0.01 & 0.01 & 0.01 & 0.01 & 0.01\\0.01 & 0.01 & 0.01 & 0.01 & 0.01 \end{bmatrix}$$

$$LT_{table} = w_{table} \cdot C^{\mathsf{T}} = \begin{bmatrix} 0.01\\ 0.01\\ 0.01 \end{bmatrix}$$

Feature vectors same for all words.

Feedforward Neural Network with Lookup Table



Note: Bias terms omitted for simplicity

Weight training

Training: Find weight matrices C, U and V such that h(X) is the **correct** answer as many times as possible.

- \rightarrow Given a set T of training examples $t_1, \dots t_n$ with **correct labels y**_i, find C, U and V such that $h(X) = \mathbf{y}_i$ for as many t_i as possible.
 - \rightarrow Computation of h(X) with forward propagation
 - \rightarrow C, U and V with error back propagation

Dot product with (trained) weight matrix

 $W = \{ \mathsf{the}, \mathsf{cat}, \mathsf{on}, \mathsf{table}, \mathsf{chair} \}$

$$w_{table} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0.02 & 0.1 & 0.05 & 0.03 & 0.01 \\ 0.15 & 0.2 & 0.01 & 0.02 & 0.11 \\ 0.03 & 0.1 & 0.04 & 0.04 & 0.12 \end{bmatrix}$$

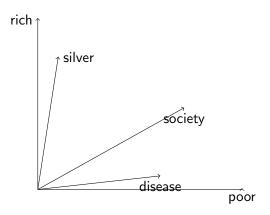
$$LT_{table} = w_{table} \cdot C^{T} = \begin{bmatrix} 0.03 \\ 0.02 \\ 0.04 \end{bmatrix}$$

Each word gets a specific feature vector

WORD EMBEDDINGS

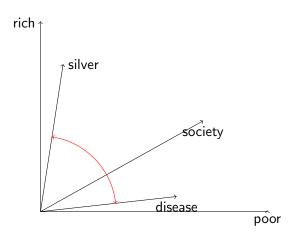
Word Embeddings

• Representation of words in vector space



Word Embeddings

- Similar words are close to each other
 - → Similarity is the cosine of the angle between two word vectors



Underlying thoughts

- Assume the equivalence of:
 - ► Two words are semantically similar.
 - ► Two words occur in similar contexts (Miller & Charles, roughly).
 - ▶ Two words have similar word neighbors in the corpus.
- Elements of this are from Leibniz, Harris, Firth, and Miller.
- Strictly speaking, similarity of neighbors is neither necessary nor sufficient for semantic similarity.
- But perhaps this is good enough.

Adapted slide from Hinrich Schütze

Learning word embeddings

Count-based methods:

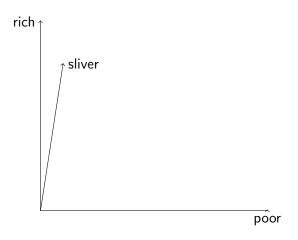
- Compute cooccurrence statistics
- Learn high-dimensional representation
- Map sparse high-dimensional vectors to small dense representation

Word cooccurrence in Wikipedia

- corpus = English Wikipedia
- cooccurrence defined as occurrence within k = 10 words of each other
 - cooc.(rich,silver) = 186
 - cooc.(poor,silver) = 34
 - cooc.(rich,disease) = 17
 - cooc.(poor,disease) = 162
 - cooc.(rich,society) = 143
 - cooc.(poor,society) = 228

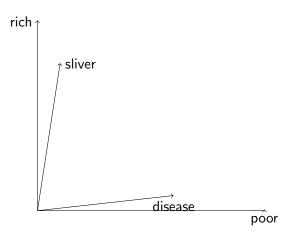
Adapted slide from Hinrich Schütze

Coocurrence-based Word Space



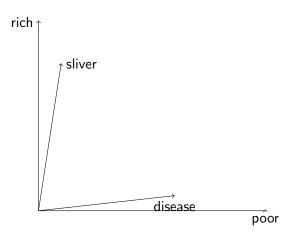
cooc.(poor,silver)=34,cooc.(rich,silver)=186

Coocurrence-based Word Space



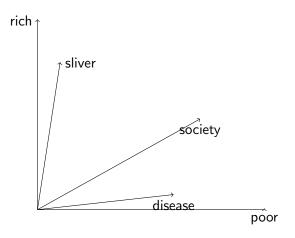
cooc.(poor,disease) = 162,cooc.(rich,disease) = 17.

Exercise



ccooc.(poor,society)=228, cooc.(rich,society)=143 How is it represented?

Coocurrence-based Word Space



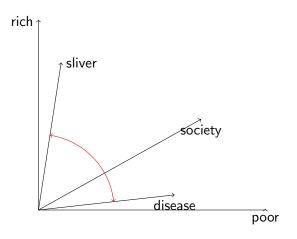
cooc.(poor,society)=228, cooc.(rich,society)=143

Dimensionality of word space

- Up to now we've only used two dimension words: rich and poor.
- ullet Do this for all possible words in a corpus o high-dimensional space
- Formally, there is no difference to a two-dimensional space with three vectors.
- Note: a word can have a dual role in word space.
 - ► Each word can, in principle, be a dimension word, an axis of the space.
 - But each word is also a vector in that space.

Adapted slide from Hinrich Schütze

Semantic similarity



Similarity is the cosine of the angle between two word vectors

Learning word embeddings

Count-based methods:

- Compute cooccurrence statistics
- Learn high-dimensional representation
- Map sparse high-dimensional vectors to small dense representation

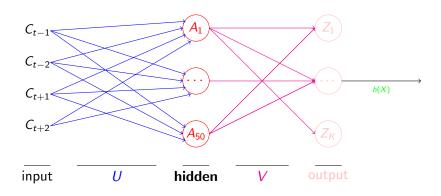
Neural networks:

- Predict a word from its neighbors
- Learn (small) embedding vectors

Word vectors with Neural Networks

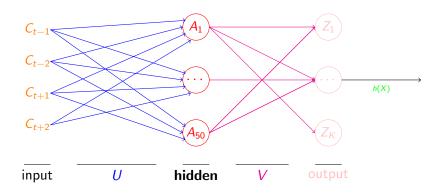
- LM Task: Given k previous words, predict the current word
 - \rightarrow For each word w in V, model $P(w_t|w_{t-1}, w_{t-2}, ..., w_{t-n})$
 - → Learn embeddings C of words
 - \rightarrow Input for task
- Task: Given k context words, predict the current word
 - → Learn embeddings C of words

Network architecture



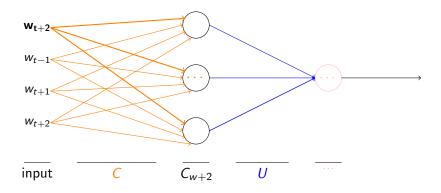
Given words w_{t-2} , w_{t-1} , w_{t+1} and w_{t+2} , predict w_t Note: Bias terms omitted for simplicity

Network architecture



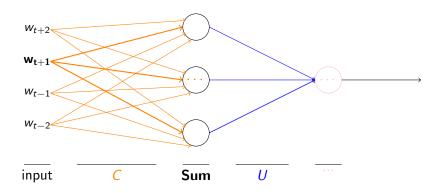
We want the context vectors \rightarrow embed words in shared space Note: Bias terms omitted for simplicity

Getting the Word Embeddings



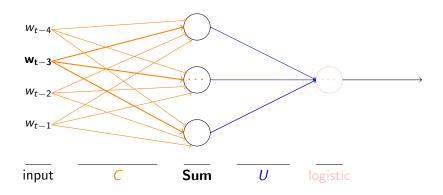
Note: Bias terms omitted for simplicity

- Remove hidden layer
- Sum over all projections



Remove hidden layer and sum over context Note: Bias terms omitted for simplicity

- Single logistic unit instead of output layer
 - → No need for distribution over words (only vector representation)
 - → Task as binary classification problem:
 - Given input and weight matrix say if w_t is current word
 - ▶ We know the correct w_t , how do we get the wrong ones?
 - → negative sampling



Remove hidden layer and sum over context Note: Bias terms omitted for simplicity

Word2Vec

- BOW model (Mikolov. 2013)
- Skip-gram model:
 - ▶ Input is w_t
 - ▶ Prediction is w_{t+2} , w_{t+1} , w_{t-1} and w_{t-2}

Applications

Semantic similarity:

- How similar are the words:
 - coast and shore; rich and money; happiness and disease; close and open
- WordSim-353 (Finkelstein et al. 2002)
 - Measure associations
- SimLex-999
 - Only measure semantic similarity

Other tasks:

 Use word embeddings as input features for other tasks (e.g. sentiment analysis, language modeling)

Recap

Cannot fit data with non-linear decision boundary with linear models

Solution: compose non-linear functions with neural networks

- → Successful in many NLP applications:
 - Language modeling
 - Learning word embeddings
- Feeding word embeddings into neural networks has proven successful in many NLP tasks
 - Sentiment analysis
 - Named Entity Recognition

Thank you!