Einführung in die Computerlinguistik
Hidden Markov Models (HMMs)

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Die Grundfassung dieses Foliensatzes wurde von Prof. Dr. Hinrich Schütze erstellt, basiert auf:

Chris Manning and Hinrich Schütze, Foundations of Statistical Natural Language Processing, MIT Press. Cambridge, MA: May 1999.
https://nlp.stanford.edu/fsnlp/

Fehler und Mängel sind ausschließlich meine Verantwortung.

Outline

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Statistical Natural Language Processing

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Definition

Statistical Natural Language Processing (StatNLP) uses methods of supervised, semisupervised and unsupervised learning to address tasks that involve written or spoken (human) language.

What does “statistical” mean?

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Adjective for “statistics”

statistics = the practice or science of collecting and analyzing numerical data

statistics vs. machine learning

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Statistical parameter estimation

an important / the most important subfield of machine learning

statistics vs. machine learning

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  - a large group of researchers that use existing statistical methods
  - a small group of researchers that do active research on machine learning methods

Recent big success story 1

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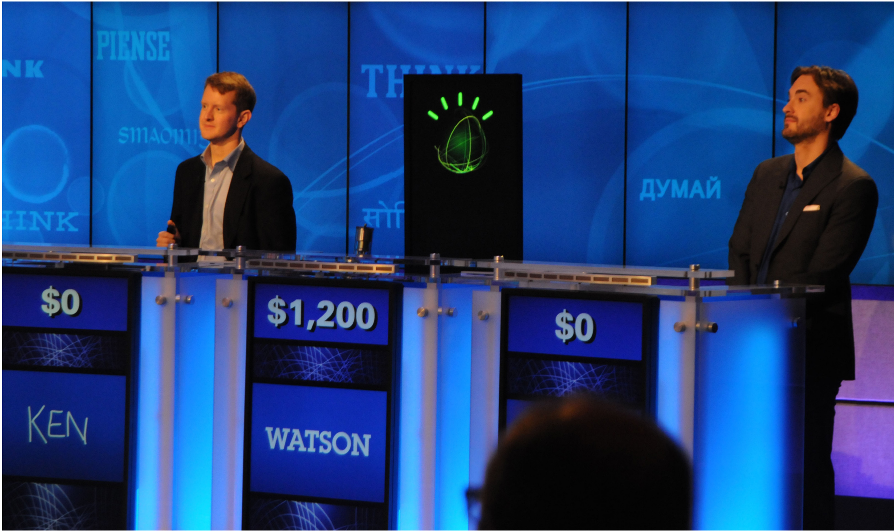

Recent big success story 2

Recent big success story 2

Recent big success story 3

Google Translate – more on this later

Outline

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Positive factor $c > 0$ does not affect argmax

$$\mathrm{argmax}_x f(x) = \mathrm{argmax}_x c \cdot f(x)$$

$$\mathrm{argmax}_x f(x) = \mathrm{argmax}_x 1/c \cdot f(x)$$

$\sum$

$$\sum_{i=m}^{i=n} f(i) = f(m) + f(m+1) + \ldots + f(n-1) + f(n)$$

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- From Axiom 2, it is obvious that $P(A) + P(\overline{A}) = 1$

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- Kolmogrov Axiom 3:
    3. If A and B are mutually exclusive (same as $P(AB) = 0$) then the probability of A or B occurring is $P(A) + P(B)$

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- Definition: $P(A|B) = \frac{P(AB)}{P(B)}$ $(P(B) > 0)$

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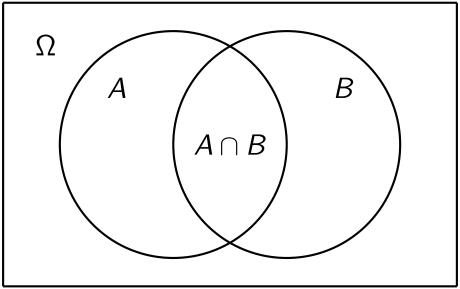

To compute $P(A|B)$: Divide the area of $A \cap B$ by the area of B.

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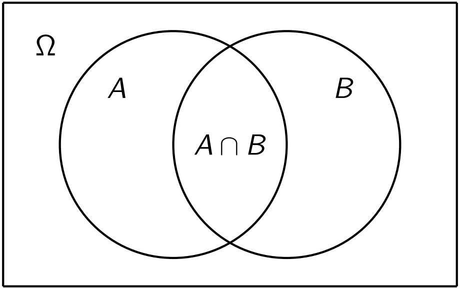

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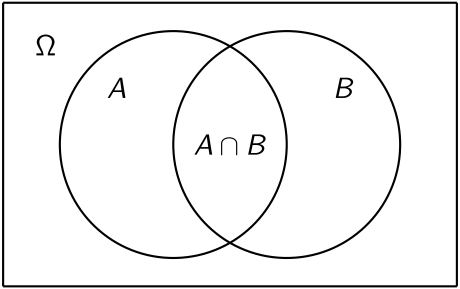

To compute $P(A|B)$: Divide the area of $A \cap B$ by the area of B.
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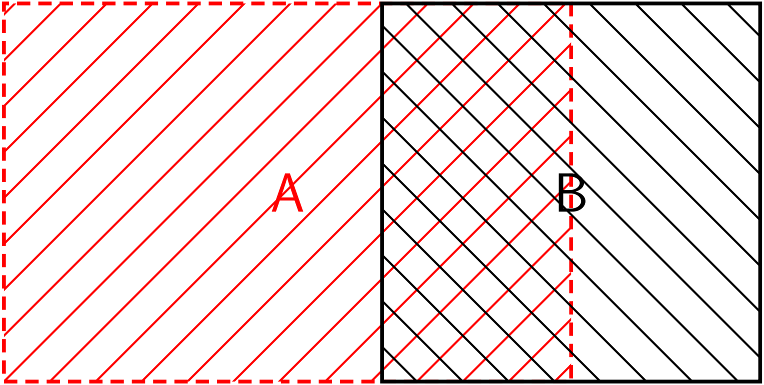
Übung



Compute $P(A|B) = P(A \cap B)/P(B)$ and
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Chain rule

$$P(X_1 X_2 X_3 \ldots X_n) =$$

$$P(X_1) \cdot P(X_2|X_1) \cdot P(X_3|X_1 X_2) \cdot \ldots \cdot P(X_n|X_1 X_2 \ldots X_{n-1})$$

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- Follows from
  $P(A) = P(AB) + P(A\overline{B}) = P(A|B)P(B) + P(A|\overline{B})P(\overline{B})$

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- Why $\approx$?

Testing for independence: Example

A = champagne, B = sparkling

Übung

Find either two independent words or two words that occur less often on the same page than expected by chance

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- Example: "book" is either a verb or a noun.
- In the context "the book" it can only be a noun.
- In the context "to book a flight" it can only be a verb.
- Part-of-speech tagging assigns to "book" the correct syntactic category in context.

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- Are all cases of part-of-speech tagging this easy?

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The following sentence is ambiguous wrt POS. Why?

The	representative	put	chairs	on	the	table
AT	NN	VBD	NNS	IN	AT	NN
article	noun	verb-d	noun-s	prep	article	noun

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In this case, finding the correct parts of speech for the sentence is more difficult.

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- It is solvable: Very high accuracy rates can be achieved (95–98% for English).
- It helps with many things you want to do with text, e.g., chunking, information extraction, question answering and parsing.

Part-of-speech tagging of tweets

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ikr	smh	he	asked	fir	yo	last
!	G	O	V	P	D	A
name	so	he	can	add	u	on
N	P	O	V	V	O	P
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Example from: Owoputi et al. (2012). Part-of-Speech Tagging for Twitter: Word Clusters and Other Advances. Tech Report. See `http://www.cs.cmu.edu/~ark/TweetNLP/`

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AT	article
BEZ	the word "is"
IN	preposition
JJ	adjective
JJR	comparative adjective
MD	modal
NN	singular or mass noun
NNP	singular proper noun
NNS	plural noun
PERIOD	. : ? !
PN	personal pronoun

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RB	adverb
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TO	the word "to"
VB	verb, base form
VBD	verb, past tense
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Tag: "Peter arrived in London on Tuesday"

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Two main sources of information

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   the words to the left and to the right

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   - Example: for a JJ/NN ambiguity in the context "AT _ VBZ", NN is much more likely than JJ.
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   - Example: "put" is much more likely to occur as a VBD than as an NN.

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- This source of information lets us do 90% correct tagging of English very easily: Just pick the most frequent tag for each word.
- For most words in English, the distribution of tags is very uneven: there is one very frequent tag and the others are rare.

Notation

Notation

w_i	the word at position i in the corpus
t_i	the tag of w_i
w^l	the l^{th} word in the lexicon
t^j	the j^{th} tag in the tag set
$C(w^l)$	the number of occurrences of w^l in the training set
$C(t^j)$	the number of occurrences of t^j in the training set
$C(t^j t^k)$	the number of occurrences of t^j followed by t^k
$C(w^l : t^j)$	the number of occurrences of w^l that are tagged as t^j

Notation: Example

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the	representative	put	chairs	on	the	table
w_1	w_2	w_3	w_4	w_5	w_6	w_7
w^5	w^{81}	w^3	w^4	w^1	w^5	w^6
AT	NN	VBD	NNS	IN	AT	NN
article	noun	verb-d	noun-s	prep	article	noun
t_1	t_2	t_3	t_4	t_5	t_6	t_7
t^{16}	t^{12}	t^2	t^9	t^3	t^{16}	t^{12}

$$
\begin{array}{lcl|lcl}
C(w^5) & = & 2 & C(w^4) & = & 1 \\
C(t^{16}) & = & 2 & C(t^2) & = & 1 \\
C(t^{16}t^{12}) & = & 2 & C(t^{12}t^2) & = & 1 \\
C(t^{16}t^2) & = & 0 & C(w^5w^{81}) & = & 1 \\
C(w^5 : t^{16}) & = & 2 & C(w^5 : t^{12}) & = & 0
\end{array}
$$

Notation: Übung

Confidence/NN in/IN the/AT pound/NN is/BEZ widely/RB expected/VBN to/TO take/VB another/AT sharp/JJ dive/NN if/IN trade/NN figures/NNS for/IN September/NNP ,/, due/JJ for/IN release/NN tomorrow/NN ,/, fail/VB to/TO show/VB a/AT substantial/JJ improvement/NN from/IN July/NNP and/CC August/NNP 's/POS near-record/JJ deficits/NNS ./. Chancellor/NNP of/IN the/AT Exchequer/NNP Nigel/NNP Lawson/NNP 's/POS restated/VBN commitment/NN to/TO a/AT firm/JJ monetary/JJ policy/NN has/VBZ helped/VBN to/TO prevent/VB a/AT freefall/NN in/IN sterling/NN over/IN the/AT past/JJ week/NN ./.

Give the values of the following: w_4, t_5, $C(w_8)$, $C(t_9)$, $C(t_1 t_2)$, $C(w_3 : t_3)$

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- Train a statistical model on the training set
  - Result: A set of parameters (= numbers) that were learned from the specific properties of the training set
- Apply statistical model to new text that we want to analyze for some task (information retrieval, machine translation etc)

Tagged training corpus/set: Example

Confidence/NN in/IN the/AT pound/NN is/BEZ widely/RB expected/VBN to/TO take/VB another/AT sharp/JJ dive/NN if/IN trade/NN figures/NNS for/IN September/NNP ,/, due/JJ for/IN release/NN tomorrow/NN ,/, fail/VB to/TO show/VB a/AT substantial/JJ improvement/NN from/IN July/NNP and/CC August/NNP 's/POS near-record/JJ deficits/NNS ./. Chancellor/NNP of/IN the/AT Exchequer/NNP Nigel/NNP Lawson/NNP 's/POS restated/VBN commitment/NN to/TO a/AT firm/JJ monetary/JJ policy/NN has/VBZ helped/VBN to/TO prevent/VB a/AT freefall/NN in/IN sterling/NN over/IN the/AT past/JJ week/NN ./.

Outline

Contents of this section

- Parameter estimation: context parameters
- Parameter estimation: bias parameters
- Greedy tagging
- Viterbi tagging

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  - Limited horizon, Markov assumption: we assume that our memory is limited to a single preceding tag.
  - Time invariance, stationary: we assume that these conditional probabilities don't change. (e.g., the same at the beginning and at the end of the sentence)

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n^{th} order Markov model

In an n^{th} order Markov model,

the tag at time t depends on the n previous tags.

- Order 0: Tag does not depend on previous tags.
- Order 1: Tag depends on immediately preceding tag.
- Order 2: Tag depends on two immediately preceding tags.
- Order 3: Tag depends on three immediately preceding tags.
- ...

(analogous for Markov model that emits words instead of tags)

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Estimate $P(\text{take}|\text{VB})$ and $P(\text{AT}|\text{IN})$

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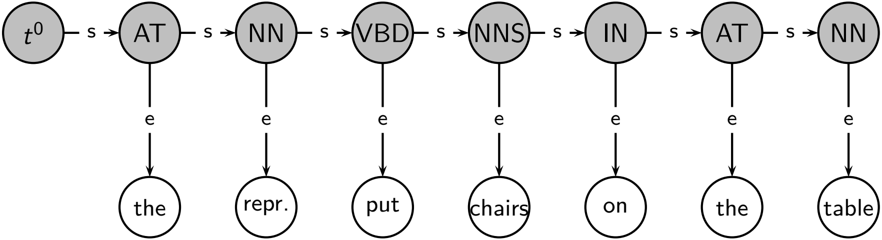
$P(w|t)$ versus $P(t|w)$

(s = sequence, e = emission)



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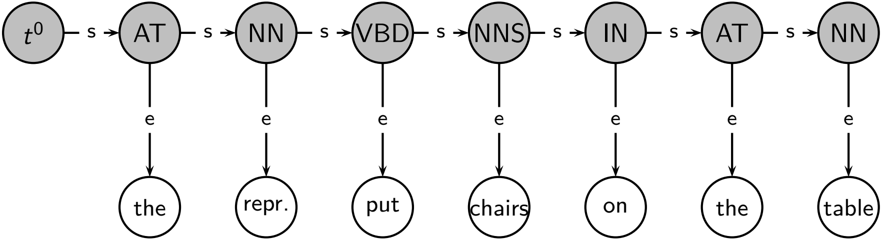

- This is a so-called "generative model".
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- At decoding time, our task is to recover the tags (= states). This model is called a "Hidden Markov Model" because we don't know the states.

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- How can we do this?

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- $t_3 = \text{VBD}$ maximizes $P(t_3|\text{NN})P(\text{put}|t_3)$

Problems with greedy tagging

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- What can go wrong with greedy tagging?
- Example?
- The representative put costs 20% more today than a month ago.

Notation (2)

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w_i	the word at position i in the corpus
t_i	the tag of w_i
$w_{i,i+m}$	the words occurring at positions i through $i+m$ (alternative notations: $w_i \cdots w_{i+m}$, $w_i, \ldots, w_{i+m}$, $w_{i(i+m)}$)
$t_{i,i+m}$	the tags $t_i \cdots t_{i+m}$ for $w_i \cdots w_{i+m}$
w^l	the l^{th} word in the lexicon
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$C(w^l)$	the number of occurrences of w^l in the training set
$C(t^j)$	the number of occurrences of t^j in the training set
$C(t^j t^k)$	the number of occurrences of t^j followed by t^k
$C(w^l : t^j)$	the number of occurrences of w^l that are tagged as t^j
T	number of tags in tag set
W	number of words in the lexicon
n	sentence length

Part-of-speech tagging: Problem statement

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- We define our goal thus: Given a sentence, find the most probable sequence of tags for this sentence.
- Formalization of this goal:

$$t_{1,n} = \text{argmax}_{t_{1,n}} P(t_{1,n}|w_{1,n})$$

Simplifying the argmax (1)

$$
\begin{aligned}
t_{1,n} &= \text{argmax}_{t_{1,n}} P(t_{1,n}|w_{1,n}) && (1)\\
&= \text{argmax}_{t_{1,n}} P(t_{0,n}|w_{1,n}) && (2)\\
&= \text{argmax}_{t_{1,n}} \frac{P(w_{1,n}|t_{0,n})P(t_{0,n})}{P(w_{1,n})} && (3)\\
&= \text{argmax}_{t_{1,n}} P(w_{1,n}|t_{0,n})P(t_{0,n}) && (4)\\
&= \text{argmax}_{t_{1,n}} [\prod_{i=1}^{n} P(w_i|t_{0,n})]P(t_{0,n}) && (5)
\end{aligned}
$$

2: dummy “start” tag; 3: Bayes; 4: positive factor doesn't affect argmax; 5: assumption: words are independent

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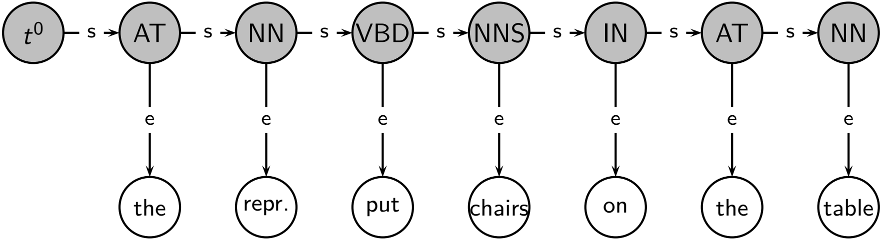

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Simplifying the argmax (2)

$$= \text{argmax}_{t_{1,n}} [\prod_{i=1}^{n} P(w_i|t_i)] P(t_{0,n}) \quad (6)$$

$$= \text{argmax}_{t_{1,n}} [\prod_{i=1}^{n} P(w_i|t_i)] [\prod_{i=1}^{n} P(t_i|t_{0,i-1})] \quad (7)$$

$$= \text{argmax}_{t_{1,n}} [\prod_{i=1}^{n} P(w_i|t_i)] [\prod_{i=1}^{n} P(t_i|t_{i-1})] \quad (8)$$

$$= \text{argmax}_{t_{1,n}} \prod_{i=1}^{n} [P(w_i|t_i) P(t_i|t_{i-1})] \quad (9)$$

7: chain rule; 8: Markov assumption; 9: $\prod_{i=1}^{n} x_i \prod_{i=1}^{n} y_i = \prod_{i=1}^{n} x_i y_i$

Simplifying the argmax (3)

$$= \quad \text{argmax}_{t_{1,n}} \prod_{i=1}^{n} [P(w_i|t_i)P(t_i|t_{i-1})] \qquad (10)$$

$$= \quad \text{argmax}_{t_{1,n}} \sum_{i=1}^{n} [\log P(w_i|t_i) + \log P(t_i|t_{i-1})] \qquad (11)$$

11: computation in log space more efficient / convenient

Outline

The most probable tag sequence (= tagging)

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What's the difficulty if you want to tag based on this?

Brute force is very inefficient

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There are $|T|^n$ different tag sequences. E.g.:

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$40^{20} = 109{,}951{,}162{,}777{,}600{,}000{,}000{,}000{,}000{,}000{,}000$

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Is there a better way?

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- ...but I only compute it once!

$P(t_i|t_{i-1})$
Example: $P(\text{VB}|\text{MD}) = 0.7968$

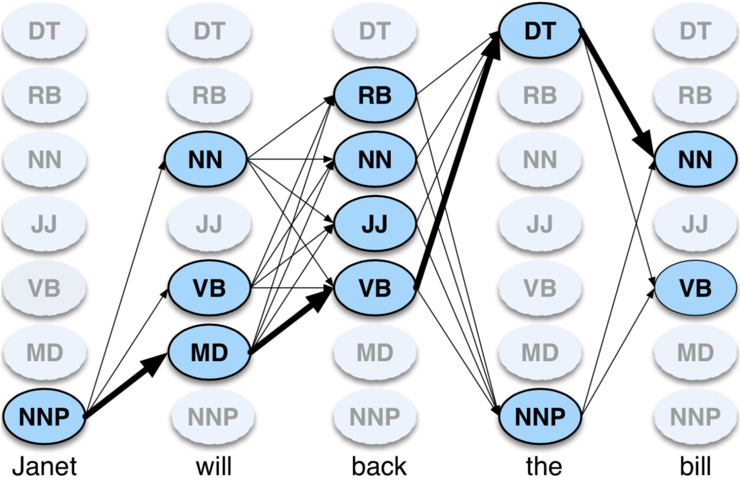
	NNP	**MD**	**VB**	**JJ**	**NN**	**RB**	**DT**
$<s>$	0.2767	0.0006	0.0031	0.0453	0.0449	0.0510	0.2026
NNP	0.3777	0.0110	0.0009	0.0084	0.0584	0.0090	0.0025
MD	0.0008	0.0002	0.7968	0.0005	0.0008	0.1698	0.0041
VB	0.0322	0.0005	0.0050	0.0837	0.0615	0.0514	0.2231
JJ	0.0366	0.0004	0.0001	0.0733	0.4509	0.0036	0.0036
NN	0.0096	0.0176	0.0014	0.0086	0.1216	0.0177	0.0068
RB	0.0068	0.0102	0.1011	0.1012	0.0120	0.0728	0.0479
DT	0.1147	0.0021	0.0002	0.2157	0.4744	0.0102	0.0017

vertical axis: t_{i-1}
horizontal axis: t_i

$P(w|t)$
Example: $P(the|\mathrm{DT}) = 0.506099$

	Janet	**will**	**back**	**the**	**bill**
NNP	0.000032	0	0	0.000048	0
MD	0	0.308431	0	0	0
VB	0	0.000028	0.000672	0	0.000028
JJ	0	0	0.000340	0	0
NN	0	0.000200	0.000223	0	0.002337
RB	0	0	0.010446	0	0
DT	0	0	0	0.506099	0

Key idea of Viterbi: Lattice



Viterbi

function VITERBI(*observations* of len T,*state-graph* of len N) **returns** *best-path*, *path-prob*

create a path probability matrix *viterbi[N,T]*
for each state s **from** 1 **to** N **do** ; initialization step
  $viterbi[s,1] \leftarrow \pi_s * b_s(o_1)$
  $backpointer[s,1] \leftarrow 0$
for each time step t **from** 2 **to** T **do** ; recursion step
  for each state s **from** 1 **to** N **do**
  $viterbi[s,t] \leftarrow \max_{s'=1}^{N} viterbi[s', t-1] * a_{s',s} * b_s(o_t)$
  $backpointer[s,t] \leftarrow \operatorname{argmax}_{s'=1}^{N} viterbi[s', t-1] * a_{s',s} * b_s(o_t)$
$bestpathprob \leftarrow \max_{s=1}^{N} viterbi[s, T]$; termination step
$bestpathpointer \leftarrow \operatorname{argmax}_{s=1}^{N} viterbi[s, T]$; termination step
bestpath ← the path starting at state *bestpathpointer*, that follows backpointer[] to states back in time
return *bestpath*, *bestpathprob*

$P(t_i|t_{i-1})$
Example: $P(\text{VB}|\text{NN}) = 0.5$

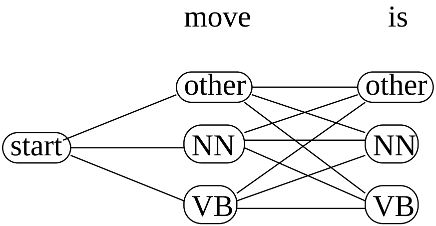
next / prev	other	NN	VB
start	0.3	0.4	0.3
other	0.2	0.2	0.6
NN	0.4	0.1	0.5
VB	0.1	0.8	0.1

vertical axis: t_{i-1}
horizontal axis: t_i

$P(w|t)$
Example: $P(bear|\text{NN}) = 0.45$

	other	NN	VB
bear	0.1	0.45	0.4
is	0.3	0.05	0.05
on	0.3	0.05	0.05
the	0.2	0.05	0.05
move	0.1	0.4	0.45

Lattice



Goal: Compute

$$\arg\max_{t_1,t_2} p(t_1, move, t_2, is) =$$

$$\arg\max_{t_1,t_2} p(t_1|start)p(move|t_1)p(t_2|t_1)p(is|t_2)$$

viterbi = vtrb
backpointer = bptr
lattice = path probability matrix

vtrb and bptr

$\text{vtrb}_j(t_i)$ is the probability of [the most probable path from 0 to j that tags word w_j with tag t_i].

$\text{bptr}_j(t_i)$ is the tag of w_{j-1} on [the most probable path from 0 to j that tags word w_j with tag t_i].

Initialization: $\text{vtrb}_0(\textit{start}) = 1$

Viterbi probabilities for the tags of the first word

$\text{vtrb}_1(\text{other}) = \text{vtrb}_0(\text{start})\ p(\text{other}|\text{start})\ p(\text{move}|\text{other}) = 1.0 * 0.3 * 0.1 = 0.03$
$\text{vtrb}_1(\text{NN}) = \text{vtrb}_0(\text{start})\ p(\text{NN}|\text{start})\ p(\text{move}|\text{NN}) = 1.0 * 0.4 * 0.4 = 0.16$
$\text{vtrb}_1(\text{VB}) = \text{vtrb}_0(\text{start})\ p(\text{VB}|\text{start})\ p(\text{move}|\text{VB}) = 1.0 * 0.3 * 0.45 = 0.135$

Viterbi probabilities for the tags of the second word (1)

$\text{vtrb}_2(\text{other}) = \max($
$\quad \text{vtrb}_1(\text{other})\ p(\text{other}|\text{other})\ p(\text{is}|\text{other}) = 0.03 * 0.2 * 0.3 = 0.0018,$
$\quad \text{vtrb}_1(\text{NN})\ p(\text{other}|\text{NN})\ p(\text{is}|\text{other}) = 0.16 * 0.4 * 0.3 = 0.0192,$
$\quad \text{vtrb}_1(\text{VB})\ p(\text{other}|\text{VB})\ p(\text{is}|\text{other}) = 0.135 * 0.1 * 0.3 = 0.00405$
$) = 0.0192$
$\quad \text{bptr}_2(\text{other}) = \text{NN}$

Viterbi probabilities for the tags of the second word (2)

$\text{vtrb}_2(\text{NN}) = \max($
$\quad \text{vtrb}_1(\text{other})\ p(\text{NN}|\text{other})\ p(\text{is}|\text{NN}) = 0.03 * 0.2 * 0.05 = 0.0003,$
$\quad \text{vtrb}_1(\text{NN})\ p(\text{NN}|\text{NN})\ p(\text{is}|\text{NN}) = 0.16 * 0.1 * 0.05 = 0.0008,$
$\quad \text{vtrb}_1(\text{VB})\ p(\text{NN}|\text{VB})\ p(\text{is}|\text{NN}) = 0.135 * 0.8 * 0.05 = 0.0054$
$) = 0.0054$
$\quad \text{bptr}_2(\text{NN}) = \text{VB}$

Viterbi probabilities for the tags of the second word (3)

$\text{vtrb}_2(\text{VB}) = \max($
$\quad \text{vtrb}_1(\text{other})\ p(\text{VB}|\text{other})\ p(\text{is}|\text{VB}) = 0.03 * 0.6 * 0.05 = 0.0009,$
$\quad \text{vtrb}_1(\text{NN})\ p(\text{VB}|\text{NN})\ p(\text{is}|\text{VB}) = 0.16 * 0.5 * 0.05 = 0.004,$
$\quad \text{vtrb}_1(\text{VB})\ p(\text{VB}|\text{VB})\ p(\text{is}|\text{VB}) = 0.135 * 0.1 * 0.05 = 0.000675$
$) = 0.004$
$\quad \text{bptr}_2(\text{VB}) = \text{NN}$

Read out path

Probability of the most likely path: $0.0192 = \max_t \mathrm{vtrb}_2(t)$
Last tag of the most likely path: other $= \arg\max_t \mathrm{vtrb}_2(t)$
First tag of the most likely path: NN $= \mathrm{bptr}_2(\text{other})$
Result:
NN other $= \arg\max_{t_1 t_2} p(t_1, \mathit{move}, t_2, \mathit{is})$

Besonders klausurrelevant

- Part-of-speech tagging, informal definition
- Part-of-speech tagging, formal definition

$$\text{argmax}_{t_{1,n}} \sum_{i=1}^{n} [\log P(w_i|t_i) + \log P(t_i|t_{i-1})]$$

- Brown part-of-speech tags
- Parameter estimation: Context

$$\hat{P}(t^k|t^j) = \frac{C(t^j t^k)}{C(t^j)}$$

- Parameter estimation: Word bias

$$\hat{P}(w^l|t^j) = \frac{C(w^l : t^j)}{C(t^j)}$$

- Order of a Markov model
- Viterbi